# Novel methods of PDEs and function spaces

University of Warsaw, 23-27.06.2025

Book of abstracts

<table>
<tr><th></th><th>23.06 Mon</th><th>24.06 Tue</th><th>25.06 Wed</th><th>26.06 Thu</th><th>27.06 Fri</th></tr>
<tr><td>09:00-09:45</td><td></td><td>Enrico Valdinoci</td><td>Luboš Pick</td><td>Frank Duzaar</td><td>Ho-Sik Lee</td></tr>
<tr><td>09:45-11:00</td><td></td><td>Carlo Alberto Antonini</td><td>Peter Hästö</td><td>Mathias Schäffner</td><td>Jihoon Ok</td></tr>
<tr><td>10:30-11:00</td><td></td><td>coffee break</td><td>coffee break</td><td>coffee break</td><td>coffee break</td></tr>
<tr><td>11:00-11:45</td><td></td><td>Erik Lindgren</td><td>Dorothee D. Haroske</td><td>Claudia Bucur</td><td>Lenka Slavíková</td></tr>
<tr><td>11:45-12:30</td><td></td><td>Lorenzo Brasco</td><td>Cristina Tarsi</td><td>Verena Bögelein</td><td>David Cruz-Uribe</td></tr>
<tr><td>12:30-13:30</td><td>lunch</td><td>lunch</td><td>lunch</td><td>lunch</td><td>lunch</td></tr>
<tr><td>13:30-14:15</td><td>Andrea Cianchi</td><td>Pêdra Andrade</td><td>Petteri Harjulehto</td><td>Jana Björn</td><td></td></tr>
<tr><td>14:15-15:00</td><td>Serena Dipierro</td><td>Minhyun Kim</td><td></td><td>Chao Zhang</td><td></td></tr>
<tr><td>15:00-15:30</td><td>coffee break</td><td>coffee break</td><td rowspan="3">poster session</td><td>coffee break</td><td></td></tr>
<tr><td>15:30-16:15</td><td>Marvin Weidner</td><td rowspan="2">contribute session</td><td rowspan="2">contribute session</td><td></td></tr>
<tr><td>16:15-17:00</td><td>Simon Nowak</td><td></td></tr>
<tr><td>Evening</td><td></td><td></td><td>dinner</td><td></td><td></td></tr>
</table>

**Contribute session**

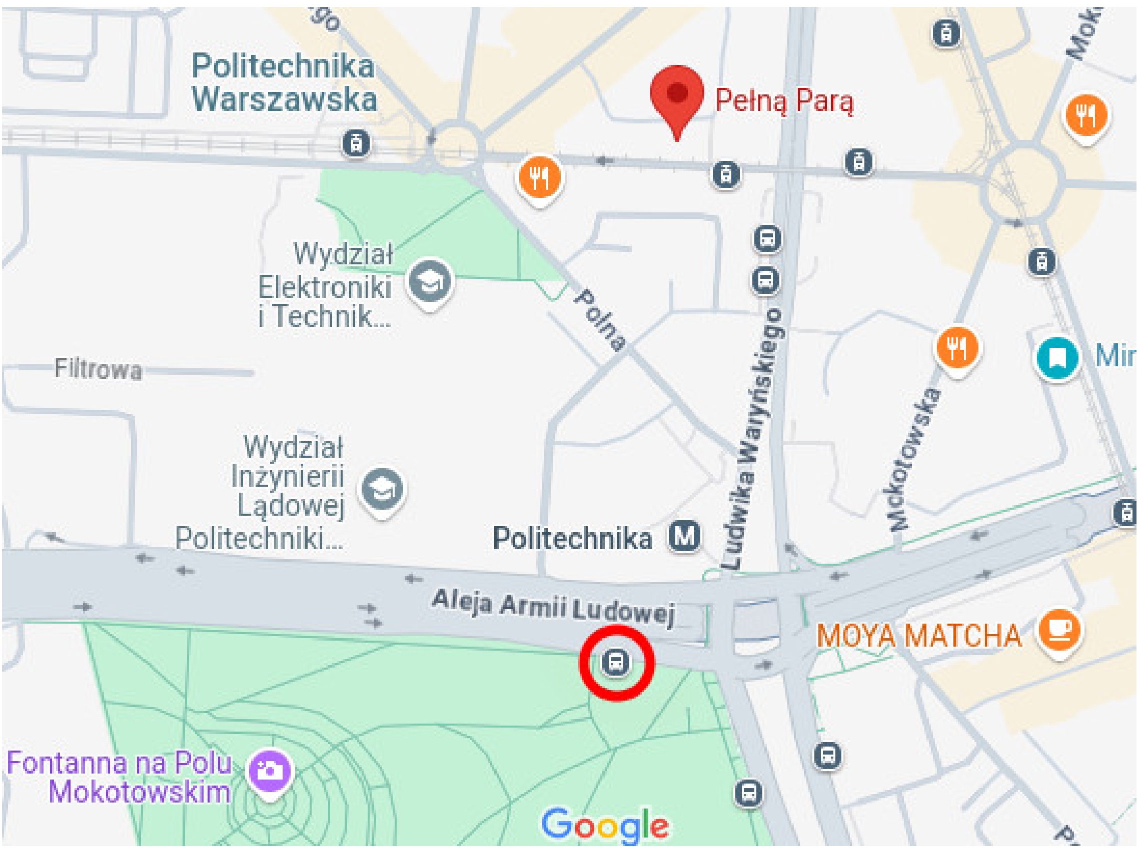
| | Tuesday | Thursday |
|---|---|---|
| 15:30-15:55 | Michał Borowski | Agnieszka Kałamajska |
| 15:55-16:20 | Leach Schätzler | Anders Björn |
| 16:20-16:45 | Filomena Feo | Julia Lenczewska |

Localization:
Faculty of Mathematics, Informatics and Mechanics
Banacha 2a, Warsaw
Room 2180

# Conference dinner

**Where**: Nowowiejska 10, https://maps.app.goo.gl/S6bibnL68m5hxEYAA
**When**: Wednesday, 8pm
**How to get there**:

- Bus from the faculty
  Line: 182, 187 or 523
  Stops: Wawelska 05 → Metro Politechnika 01
  https://maps.app.goo.gl/tA2C8Zoyr9HbKAY8A
- Bus from the hotel
  Line: 182, 187 or 523
  Stops: Dworzec Zachodni 01 → Metro Politechnika 01
  https://maps.app.goo.gl/37Q6qZnWBdiEzCmJ6




Pin: dinner's localization
Circle: recommended bus-stop

# Poster session

- **A Generalized Version of the Lions-type Lemma**
  Magdalena Chmara
- **On the Cauchy problem for hyperdissipative Navier-Stokes equations with initial data in Morrey smoothness spaces**
  Romaric Kana Nguedia
- **Equivalence between superharmonic functions and locally renormalized solutions**
  Ying Li
- **Boundary Regularity for a Fully Nonlinear Free Transmission Problem**
  David Stolnicki
- **Fractional fast diffusion with initial data a Radon measure**
  Jorge Ruiz Cases

# Regularity Estimates for Degenerate Fully Nonlinear Elliptic Equations

*Pêdra Andrade*
University of Lisbon, Portugal

In this talk, we present recent advances in regularity estimates for viscosity solutions of degenerate fully nonlinear elliptic equations. Such equations arise in geometric and physical contexts where uniform ellipticity fails, leading to significant analytical challenges. We establish Hölder regularity results using approximation methods and the Crandall–Ishii–Lions maximum principle.

# Regularization of weakly regular domains and applications to boundary value problems

*Carlo Alberto Antonini*
Istituto Nazionale di Alta Matematica, Italy

The regularity of the reference domain in a boundary value problem plays a crucial role in determining the global regularity of the solution. While classical results assume smooth domains, namely of class $C^2$, many regularity results remain valid even when the domain lacks smoothness, provided it satisfies certain weaker geometric or analytic conditions.
Nevertheless, the proof of such results often relies on a regularization procedure, where the reference domain is approximated by a sequence of smoother ones which preserve the key geometric and analytic features of the original one.
The goal of this talk is to present various regularization techniques for different classes of non-smooth bounded domains of $\mathbb{R}^n$. We will discuss methods tailored to specific types of irregular boundaries, with examples including convex domains, sets of finite perimeter and domains whose boundaries exhibit Sobolev-type regularity.

# Uniqueness and nonuniqueness of p-harmonic Green functions

*Jana Björn*
Linköping University, Sweden

We define $p$-harmonic Green functions in domains in metric spaces and give a new sufficient condition for their uniqueness. An example in weighted $\mathbb{R}^n$ is provided, showing that the range of new $p$'s, for which uniqueness holds, can be a nondegenerate interval. In the opposite direction, we give the first example showing that uniqueness can fail in metric spaces, even for $p = 2$.

# Regularity of $(s,p)$-harmonic functions

*Verena Bögelein*
University of Salzburg, Austria

We report on higher Sobolev and Hölder regularity results for local weak solutions of the fractional $p$-Laplace equation of order $s \in (0,1)$ with $1 < p < \infty$. The relevant estimates are stable when the fractional order $s$ reaches 1, and the known Sobolev regularity estimates for weak solutions of the local $p$-Laplace equation are recovered. As an application we establish Calderòn-Zygmund type estimates at the gradient level for the associated fractional $p$-Poisson equation. The talk is based on joint work with Frank Duzaar (Salzburg), Kristian Moring (Salzburg), Naian Liao (Salzburg), Giovanni Molica Bisci (Urbino), and Raffaella Servadei (Urbino).

# Sharp Poincaré-Sobolev inequalities in Steiner symmetric sets

*Lorenzo Brasco*
University of Ferrara, Italy

We discuss existence of extremals for sharp Poincaré-Sobolev inequalities in a class of (possibly unbounded) open sets. The relevant embedding is continuous but may fail to be compact. We also present some properties of these extremals, notably their decay properties at infinity.
Some of the results presented have been obtained in collaboration with Luca Briani (Munich) and Francesca Prinari (Pisa).

# S-minimal functions: existence and continuity

*Claudia Bucur*
University of Milan, Italy

We discuss a nonlocal fractional problem that serves as a natural nonlocal counterpart to the classical least gradient problem. Our approach establishes the existence of minimizers by using their relationship with nonlocal minimal surfaces. We also explore the continuity properties of these minimizers and provide a weak formulation of the problem. The results presented are part of a joint work with S. Dipierro, L. Lombardini, J. Mazón, and E. Valdinoci.

# Second-order regularity for elliptic systems involving the symmetric gradient

*Andrea Cianchi*
University of Florence, Italy

We deal with elliptic systems of p-Laplacian type depending on the symmetric gradient of solutions. The optimal Sobolev regularity of the associated stress field for merely square-integrable right-hand sides is discussed. This is joint work with L.Behn and L.Diening.

# Solutions to degenerate elliptic equations: existence, uniqueness, boundedness

*David Cruz-Uribe*
University of Alabama, USA

In this talk we will discuss recent work on degenerate elliptic equations. We are interested in solutions to the Dirichlet problem

$$\begin{cases} Lu = f + \mathbf{T}'\mathbf{g}, & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases}$$

where $L$ is the linear operator with lower order terms,

$$Lu = -v^{-1}\operatorname{div}(Q\,\nabla u) + \mathbf{H}\cdot\mathbf{R}u + \mathbf{S}'\mathbf{G}u + Fu.$$

Here, $\Omega \subset \mathbb{R}^n$ is a bounded domain, $v \in L^1(\Omega)$, $Q$ is an $n \times n$ self-adjoint, positive semidefinite, measurable matrix function such that $|Q(x)|_{\text{op}} \le kv(x)$, $\mathbf{R}$, $\mathbf{S}$, $\mathbf{T}$ are $N$-tuples of vector fields satisfying a degenerate subuniticity condition, $\mathbf{S}'$, $\mathbf{T}'$ are their formal adjoints, and $f$, $\mathbf{g}$, $F$, $\mathbf{G}$, $\mathbf{H}$ are in $L^2(v,\Omega)$.
For this project we follow an approach introduced by Sawyer and Wheeden, where, rather than working with specific $v$ and $Q$, we develop an abstract theory where we assume the existence of a global degenerate Sobolev inequality. We consider three cases: a Sobolev inequality with gain in the scale of Lebesgue spaces:

$$\left(\int_\Omega |\varphi|^{2\sigma} v\,dx\right)^{\frac{1}{2\sigma}} \le \left(\int_\Omega |\sqrt{Q}\,\nabla\varphi|^2\,dx\right)^{\frac{1}{2}}, \tag{1}$$

where $\sigma > 1$ and $\varphi \in Lip_0(\Omega)$; a Sobolev inequality with gain in the scale of Orlicz spaces:

$$\|\varphi\|_{L^A(v,\Omega)} \le \left(\int_\Omega |\sqrt{Q}\,\nabla\varphi|^2\,dx\right)^{\frac{1}{2}}, \tag{2}$$

where $A(t) = t^2\log(e+t)^\sigma$, $\sigma > 0$; or a Sobolev inequality without gain, that is, inequality (1) with $\sigma = 1$. For each of these assumptions in turn, we give integrability conditions on the coefficients for there to exist a unique solution to the Dirichlet problem. In each case, these integrability conditions depend on the gain in the degenerate Sobolev inequality.
We next consider the existence of bounded solutions to the principal part of the equation, that is,

$$Lu = -v^{-1}\operatorname{div}(Q\,\nabla u) = f.$$

We generalize the classical results due to Trudinger, de Giorgi, Serrin, Alexandrov, and others, by giving integrability conditions on $f$ so that solutions of this equation are bounded. In the classical case ($v = 1$, $Q$ uniformly elliptic) the assumption on $f$ is that $f \in L^q(\Omega)$, where $q > \frac{n}{2} = (\frac{n}{n-2})'$, the dual of the gain in the classical Sobolev inequality. We improve this result in two ways: we show the corresponding results assuming the existence of a degenerate Sobolev inequality with gain in the scale of Lebesgue spaces or Orlicz spaces, and we show that $f$ can be taken in an Orlicz space very close to the threshold space $L^{\sigma'}(v, \Omega)$. In the case where we have a Sobolev inequality without gain, we show that solutions are exponentially integrable.
Finally, if time permits, we will consider the existence of the degenerate Sobolev inequalities (1) and (2). We show how techniques adapted from the theory of extrapolation in harmonic analysis can yield such inequalities with only integrability assumptions on $Q$ and $v$.

This work is joint with Scott Rodney, Francis MacDonald, Canada, and Şeyma Çetin, Feyza Elif Dal, Yusuf Zeren, Turkey. This project is funded by the National Science Foundation of the United States, NSERC, Canada, and TUBITAK, the Technological Research Council of T¨urkiye.

# Boundary behavior of solutions to fractional elliptic problems

*Serena Dipierro*
University of Western Australia, Australia

Solutions of nonlocal equations typically depend rather significantly on their values outside of a given region of interest and, in this sense, it is often convenient to assume "global" conditions to deduce "local" results. In this talk, we present instead a Hopf Lemma for solutions to some integro-differential equations that does not assume any global condition on the sign of the solutions. We also show that non-trivial radial solutions cannot have infinitely many zeros accumulating at the boundary. This is a joint work with Nicola Soave (University of Turin) and Enrico Valdinoci (University of Western Australia).

# Parabolic PDEs with Dynamic Data under a Bounded Slope Condition

*Frank Duzaar*
University of Salzburg, Austria

We address the Cauchy–Dirichlet problem for a class of evolutionary partial differential equations of the form

$$\partial_t u - \operatorname{div}_x \nabla_\xi f(\nabla u) = 0$$

in a space-time cylinder $\Omega_T = \Omega \times (0, T)$, where the time dependent boundary data $g \colon \partial_{\mathcal{P}}\Omega_T \to \mathbb{R}$ are prescribed on the parabolic boundary. We establish the existence of Lipschitz-continuous solutions under minimal regularity assumptions on the data.
The main novelty of this work lies in introducing a time-dependent version of the classical bounded slope condition. Specifically, we assume that for each fixed time $t \in [0, T)$, the spatial trace $g(\cdot, t)$ admits supporting hyperplanes along $\partial\Omega$ with slopes that vary in time but remain uniformly bounded. This flexible geometric condition allows us to accommodate genuinely time-dependent boundary data. The proof relies on the construction of refined upper and lower barriers adapted to the parabolic setting, which is a key innovation in our approach.
This is joint work with Giulia Treu (University of Padova) and Verena Bögelein (University of Salzburg).

## Bounded variation spaces with generalized Orlicz growth related to image denoising

*Petteri Harjulehto*
University of Helsinki, Finland

I discuss about bounded variation spaces with generalized Orlicz growth, which are motivated by the image denoising problem and the undesirable stair-casing effect of the total variation method. The setup covers earlier variable exponent and double phase models. I introduce the norm and modular of the new space and derive a formula for the modular in terms of the Lebesgue decomposition of the derivative measure and a location dependent recession function. The modular can be obtained as the Gamma-limit of uniformly convex approximating energies.
The talk is based on my joint works with Michela Eleuteri (Modena) and Peter Hästö (Helsinki).

## Mapping properties of Fourier transforms in function spaces, some recent results

*Dorothee D. Haroske*
Friedrich Schiller University, Jena, Germany

We study continuous and compact mappings generated by the Fourier transform between distinguished Besov spaces $B^s_{p,p}(\mathbb{R}^n)$, $1 \leq p \leq \infty$, and between Sobolev spaces $H^s_p(\mathbb{R}^n)$, $1 < p < \infty$. Here we rely mainly on wavelet expansions, duality and interpolation of corresponding (unweighted) spaces, and (appropriately extended) Hausdorff-Young inequalities. The degree of compactness will be measured in terms of entropy numbers and approximation numbers, now using the symbiotic relationship to weighted spaces. We can also characterise the situation when the Fourier transform acts as a nuclear operator.

This is joint work with Leszek Skrzypczak (Poznań) and Hans Triebel (Jena).

## Mean oscillation conditions for nonlinear equation and regularity results

*Peter Hästö*
University of Helsinki, Finland

I will present results from a recent eponymous preprint with Mikyoung Lee and Jihoon Ok. We consider general nonlinear elliptic equations of the form

$$\operatorname{div} A(x, Du) = 0 \quad \text{in } \Omega,$$

where $A : \Omega \times \mathbb{R}^n \to \mathbb{R}^n$ satisfies a quasi-isotropic $(p, q)$-growth condition. We establish sharp and comprehensive mean oscillation conditions on $A(x, \xi)$ with respect to the $x$ variable to obtain $C^1$- and $W^{1,s}$-regularity results. The results provide new conditions even in the standard $p$-growth case with coefficient $\operatorname{div}(a(x)|Du|^{p-2}Du) = 0$. Also included are variable exponent growth with and without perturbation as well as borderline double-phase growth and double-phase growth with coefficient, including coefficients and exponents of Sobolev–Lorentz type.
This is joint work with Leszek Skrzypczak (Poznań) and Hans Triebel (Jena).

# Recent advances in nonlocal nonlinear potential theory

*Minhyun Kim*
Hanyang University, Korea

In this talk, I will present several recent results on nonlocal nonlinear potential theory. Nonlocal nonlinear potential theory is the study of harmonic functions with respect to nonlocal nonlinear operators modeled on the fractional $p$-Laplacian. We will study local and boundary behavior of harmonic functions. The main topics include the removability theorem, isolated singularity theorem, boundary regularity, Wiener criterion, and Green function estimates. This talk is based on joint works with Anders Bjr̈n, Jana Björn, Ki-Ahm Lee, Se-Chan Lee, and Marvin Weidner.

# Nonlocal Meyers' example

*Ho-Sik Lee*
Bielefeld University, Germany

In [Meyers '63], Meyers showed that the gradient of solutions to 2nd order elliptic equations with merely uniformly elliptic coefficients exhibit only limited integrability. While higher integrability of the gradient can be achieved by imposing additional oscillation controlling conditions (e.g., Hölder, Cordes-Nirenberg, VMO, small BMO) on the coefficients, in [Balci, Diening, Giova, Passarelli di Napoli '22], the authors recently established the higher integrability under the sharp small log-BMO condition on the coefficients, where the sharpness is confirmed by a Meyers type counterexample.
This work addresses analogous questions for nonlocal equations: Firstly, as [Meyers '63], are conditions on the coefficients necessary for higher regularity of the solution? Secondly, as [BDGP '22], can a possibly sharp, minimal assumption of the coefficients be identified to guarantee such regularity?
We provide affirmative answers to both questions by constructing a nonlocal analog of Meyers' example. The result is based on a detailed Fourier analysis and a rigorous distributional framework. Our example is robust as the order of the nonlocal equation converges to 2, which is an order of the local (2nd order) elliptic equation. We also provide analogous examples of nonlocal equations defined with Riesz fractional derivatives.

# A Hardy-Morrey inequality

*Erik Lindgren*
KTH, Stockholm, Sweden

Morrey's celebrated inequality implies the Hölder continuity of a function whose gradient is sufficiently integrable. I will discuss an inequality involving the distance function that is a natural variant of Morrey's inequality. Bounds on the optimal constant and the question of existence of extremals will be my focus. This is joint work with Simon Larson (Chalmers) and Ryan Hynd (UPenn).

# Partial regularity in nonlocal systems

*Simon Nowak*
Bielefeld University, Germany

The theory of partial regularity for elliptic systems replaces the classical De Giorgi-Nash-Moser theory for scalar equations, asserting that solutions are regular outside of an in general non-empty negligible closed subset called the singular set. The local theory was initiated by Giusti & Miranda and Morrey, in turn relying on De Giorgi's seminar ideas in the context of minimal surfaces. I will present several extensions of the classical local partial regularity theory to nonlocal integro-differential systems along with some general tools for proving $\varepsilon$-regularity theorems in nonlocal settings. This is joint work with Cristiana De Filippis and Giuseppe Mingione

# Nonlocal equations with kernels of general order

*Jihoon Ok*
Sogang University, Seoul, Korea

We consider a broad class of nonlinear integro-differential equations with a kernel whose differentiability order is described by a general function $\phi$. This class includes not only the fractional $p$-Laplace equations, but also borderline cases when the fractional order approaches 1. Under mild assumptions on $\phi$, we establish sharp Sobolev-Poincaré type inequalities for the associated Sobolev spaces. Using these inequalities, we prove Hölder regularity and Harnack inequalities for weak solutions to such nonlocal equations. All the estimates in our results remain stable as the associated nonlocal energy functional approaches its local counterpart. This is a joint work with Kyeong Song from Korea Institute for Advanced Study (KIAS).

# Continuous and compact fractional Orlicz-Sobolev embeddings on domains

*Luboš Pick*
Charles University, Prague, Czechia

Necessary and sufficient conditions will be offered for compact embeddings of fractional Orlicz-Sobolev spaces into Orlicz spaces, or into rearrangement-invariant spaces, on bounded Lipschitz domains. The optimal Orlicz target space and the optimal rearrangement-invariant target space, both among Orlicz spaces, and within a wider class of rearrangement-invariant spaces for merely continuous embeddings will also be exhibited. This is a joint work with Angela Alberico, Andrea Cianchi and Lenka Slavíková.

## Regularity for monotone operators and applications to homogenization

*Mathias Schäffner*
Martin Luther University, Halle-Wittenberg, Germany

I will present some regularity results for solutions of nonlinear elliptic equations. Assuming only mild monotonicity and growth conditions on the operator, we establish higher integrability for the gradient of solutions.

The main motivation for this is that these monotonicity and growth conditions are stable under homogenization. As an application, we obtain uniform Calderon-Zygmund and Lipschitz estimates (on large scales) for p-Laplacian type operators with rapidly oscillating periodic coefficients. Based on joint work with Lukas Koch.

## Strongly nonlinear Robin problems for harmonic and polyharmonic functions in the half-space

*Lenka Slavíková*
Charles University, Prague, Czechia

In this talk, we discuss existence and global regularity results for boundary-value problems of Robin type for harmonic and polyharmonic functions in $n$-dimensional half-spaces. The Robin condition on the normal derivative on the boundary of the half-space is prescribed by a nonlinear function $\mathcal{N}$ of the relevant harmonic or polyharmonic functions. General Orlicz type growths for the function $\mathcal{N}$ are considered. For instance, functions $\mathcal{N}$ of classical power type, their perturbations by logarithmic factors, and exponential functions are allowed. New sharp boundedness properties in Orlicz spaces of some classical operators from harmonic analysis, of independent interest, are critical for our approach. This is a joint work with Andrea Cianchi and Gael Y. Diebou.

## Planar Schrödinger Poisson equations with critical growth and related inequalities

*Cristina Tarsi*
University of Milan, Italy

The Schr¨odinger-Poisson equation has been first introduced in dimension $N = 3$ in 1954 by Pekar to describe quantum theory of a polaron at rest, and then applied by Choquard in 1976 as an approximation to the Hartree-Fock theory of one-component plasma. It has been extensively studied in the higher dimensional case $N \geq 3$, due to the richness of plenty of applications and to the new mathematical challenges related to nonlocal problems. On the other hand, the literature is not abundant for the planar case $N = 2$, due to the presence of a sign-changing and unbounded logarithmic integral kernel, which demands for new functional settings where implementing the variational approach. We review here some recent results on this topic and on some new related inequalities.

# Sheet happens (but only as the root of $1 - s$)

*Enrico Valdinoci*
University of Western Australia, Australia

We discuss the regularity properties of two-dimensional stable $s$-minimal surfaces, presenting a robust regularity estimate and an optimal sheet separation bound, according to which the distance between different connected components of the surface must be at least the square root of $1 - s$.

# Boundary regularity for nonlocal equations

*Marvin Weidner*
University of Barcelona, Spain

There are significant differences between local and nonlocal problems when it comes to the boundary behavior of solutions. For instance, it is a well known fact that $s$-harmonic functions (i.e. solutions to nonlocal elliptic equations governed by the fractional Laplacian) are in general not better than $C^s$ up to the boundary.
As a consequence, in recent years there has been a huge interest in the boundary behavior of solutions to nonlocal equations. By now, the boundary regularity is well understood for the fractional Laplacian and for $2s$-stable nonlocal operators, however very little is known about the natural class of nonlocal operators with inhomogeneous kernels.
In this talk, I will present recent progress on the study of the inhomogeneous case, achieved in collaboration with Xavier Ros-Oton.

# Some recent results for non-local problems involving the 1-Laplacian operator

*Chao Zhang*
Harbin Institute of Technology, China

This talk concerns the nonlocal problems driven by 1-Laplacian and $(1, p)$-Laplacian operators. An existence result is presented for a fractional 1-Laplacian parabolic equation with $L^1$ data. Particular attention is given to the regularity of weak solutions to a nonlocal $(1, p)$-Laplacian elliptic equation, where almost Lipschitz continuity is established. The main analytical tools are the finite difference quotient technique, suitable energy methods and tail estimates. Based on joint work with D. Li and Y. Li.

# Short talks

## Sphericalizations of metric measure spaces

*Anders Björn*
Linköping University, Sweden

Sphericalizations are a way to deform the metric of an unbounded metric spaces so that it becomes bounded. Bonk–Kleiner (2002) gave the first definition for a general metric space. When equipped with a suitable measure, this leads to preservation of the doubling property, Poincaré inequalities and $p$-harmonic functions, with further applications.
In this talk I will discuss these results, and also a new approach which is more suitable for preservation of the Besov (fractional Sobolev) energy.

## Approximation in Variational Problems

*Michał Borowski*
University of Warsaw

We shall describe the problem of approximating functions by regular ones in the context of energy expressed by some variational functional. The existence of such an approximation implies, in particular, the absence of Lavrentiev's phenomenon. Such an approximation is possible upon imposing certain structural conditions on the functional's integrand. We shall briefly overview those conditions in special instances of functionals, such as ones of double phase or variable exponent type, and then proceed with explaining new, general ones. The talk shall describe joint work with Pierre Bousquet, Iwona Chlebicka, Benjamin Lledos, and Błażej Miasojedow.

## The asymptotic behaviour for solutions to an anisotropic diffusion equation in the slow diffusion regime

*Filomena Feo*
Dipartimento di Ingegneria, Università degli Studi di Napoli "Parthenope"

In this talk, we will present some recent results concerning the nonnegative solutions of the following anisotropic equation

$$u_t = \sum_{i=1}^{N} (u^{m_i})_{x_i x_i} \qquad \text{in} \qquad \mathbb{R}^N \times (0, +\infty)$$

with $N \geq 2$ and $m_i > 0$ for $i = 1, ..., N$. We focus on the slow ($m_i > 1$ for all $i$) diffusion in all directions. We examine, in particular, the existence and uniqueness of a self-similar fundamental solution and the asymptotic behavior of nonnegative solutions of the Cauchy problem with $L^1$ initial data. Based on some recent joint papers with J. L. Vázquez and B. Volzone.

# Density Results and Trace Operator for Weighted Dirichlet and Sobolev Spaces Defined on the Half-line

*Agnieszka Kałamajska*
University of Warsaw

We give an analytic description for the completion of $C_0^\infty(\mathbb{R}_+)$ in the weighted Dirichlet space

$$D^{1,p}(\mathbb{R}_+,\omega) := \left\{u : \mathbb{R}_+ \to \mathbb{R} \mid u \text{ is locally absolutely continuous on } \mathbb{R}_+ \text{ and } \|u'\|_{L^p(\mathbb{R}_+,\omega)} < \infty\right\},$$

for a given continuous positive weight $\omega$ defined on $\mathbb{R}_+$, where $1 < p < \infty$. The conditions are described in terms of modified variants of the $B_p$ conditions due to Kufner and Opic from 1984.
We also propose applications of our results to: obtaining new variants of the Hardy inequality; interpretation of boundary value problems in ODEs; complex interpolation theory dealing with weighted Dirichlet spaces; and deriving new Morrey-type embedding theorems for our Dirichlet space.
Similar results are obtained for the weighted Sobolev space

$$W^{1,p}(\mathbb{R}_+,t^\beta) = \left\{u \in L^p(\mathbb{R}_+,t^\beta) \mid u' \in L^p(\mathbb{R}_+,t^\beta)\right\},$$

where $\beta \in \mathbb{R}$.
The talk will be based on results obtained together with Claudia Capone and Radosław Kaczmarek.

# Hardy inequalities and nonlocal capacity

*Julia Lenczewska*
Wrocław University of Science and Technology

We introduce and study capacities related to nonlocal Sobolev spaces, with focus on spaces corresponding to zero-order nonlocal operators. In particular, we prove Hardy-type inequalities to obtain Sobolev embeddings and use them to estimate the nonlocal capacities of a ball. The talk is based on a joint work with Tomasz Grzywny.

# Doubly nonlinear systems in noncylindrical domains

*Leah Schätzler*
Aalto University

In this talk, we discuss existence and basic regularity results for doubly nonlinear systems of the type

$$\partial_t\big(|u|^{q-1}u\big) - \operatorname{div}\left(|Du|^{p-2}Du\right) = 0$$

with parameters $q \in (0,\infty)$ and $p \in (1,\infty)$ in a bounded noncylindrical domain $E \subset \mathbb{R}^n \times [0,T)$. Merely assuming that $\mathcal{L}^{n+1}(\partial E) = 0$, we have the existence of variational solutions $u \in L^\infty\big(0,T;L^{q+1}(E,\mathbb{R}^N)\big)$. If $E$ does not shrink too fast, the power $|u|^{q-1}u$ of the solution $u$ constructed in the first step admits a distributional time derivative. Moreover, under suitable conditions on $E$, $u$ is continuous with respect to time.
The talk is based on joint work with Christoph Scheven, Jarkko Siltakoski, and Calvin Stanko.