Diffusion in Warsaw

University of Warsaw, 09-12.09.2025

Book of abstracts

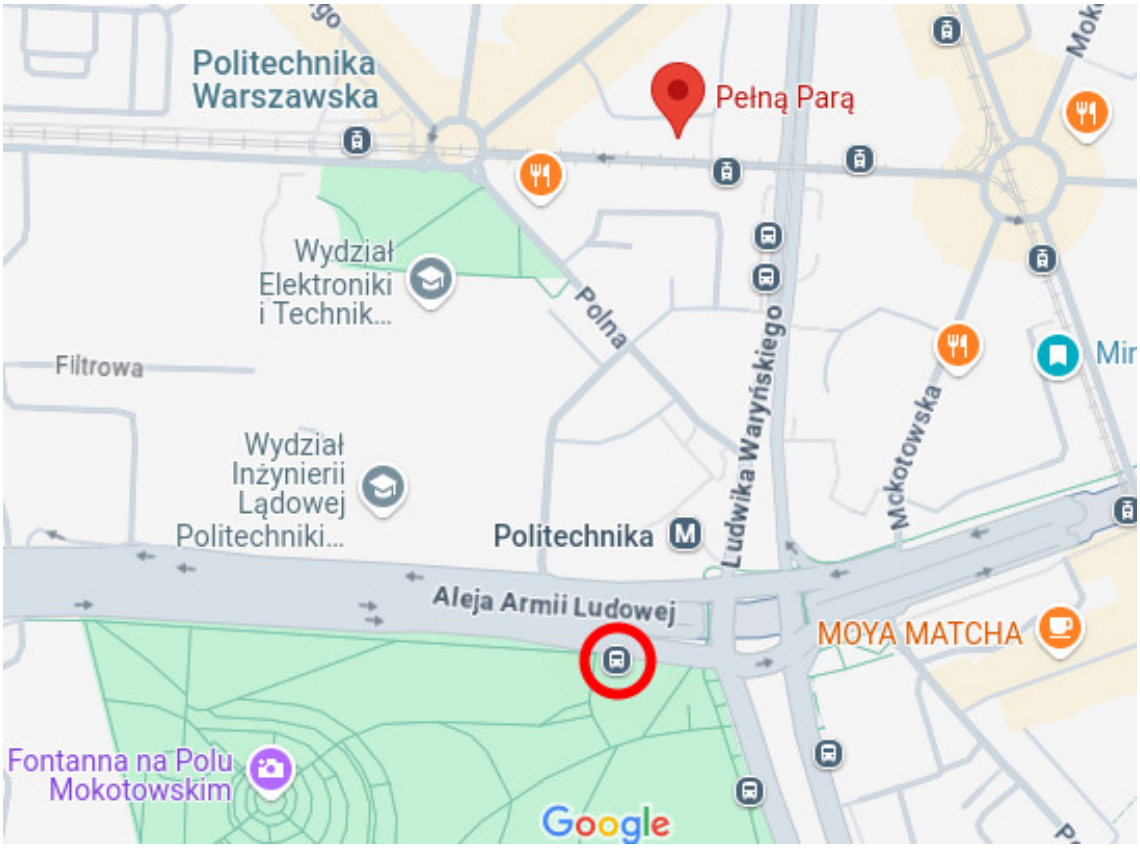
9.09 Tue	10.09 Wed	11.09 Thu	12.09 Fri
	Filippo Santambrogio 09:00-09:45	Benjamin Jourdain 09:00-09:45	Juan Luis Vazquez 09:00-09:45
	David Gómez-Castro 09:45-10:30	Katharina Schuh 09:45-10:30	Christoph Scheven 09:45-10:30
	coffee break *10:30-11:00*	*coffee break* *10:30-11:00*	*coffee break* *10:30-11:00*
	Markus Schmidtchen 11:00-11:45	Pierre Monmarché 11:00-11:45	Naian Liao 11:00-11:45
	Espen Jakobsen 11:45-12:30	Antonio Esposito 11:45-12:30	Miroslav Bulíček 11:45-12:30
lunch *12:30-13:30*	*lunch* *12:30-13:30*	*lunch* *12:30-13:30*	*lunch* *12:30-13:30*
Grzegorz Karch 13:30-14:15	Łukasz Płócienniczak 13:30-14:15	Matt Jacobs 13:30-14:15	José Miguel Urbano 13:30-14:15
Diana Stan 14:15–15:00	poster session 14:40-15:35	Matej Benko 14:15–14:40	Pierre-Cyril Aubin-Frankowski 14:15–14:40
		coffee break *14:40-15:10*	*coffee break* *14:40-15:10*
coffee break *15:00-15:30*		Tomasz Klimsiak 15:10-15:35	Dariusz Wrzosek 15:10-15:35
		Artur Rutkowski 15:35-16:00	Karol Bołbotowski 15:35-16:00

Localization:
Faculty of Mathematics, Informatics and Mechanics
Banacha 2a, Warsaw
Room 2180

Conference dinner

Where: Nowowiejska 10, https://maps.app.goo.gl/S6bibnL68m5hxEYAA
When: 20:00, Thursday, 11.09.2025
How to get there:

- Bus from the faculty
  Line: 182, 187 or 523
  Stops: Wawelska 05 → Metro Politechnika 01
  https://maps.app.goo.gl/tA2C8Zoyr9HbKAY8A
- Bus from the hotel
  Line: 182, 187 or 523
  Stops: Dworzec Zachodni 01 → Metro Politechnika 01
  https://maps.app.goo.gl/37Q6qZnWBdiEzCmJ6



Pin: dinner's localization
Circle: recommended bus-stop

Poster session

- **Improved equilibration rates to self-similarity for strong solutions of a thin-film equation**
  Mario Bukal
- **Harnack's inequalities for a nonlinear parabolic equation in non-divergence form**
  Juha Tapio Kurkinen
- **On a mathematical model for tissue regeneration**
  Nishith Mohan
- **On Cauchy Problems for Stretched Non-Local Pearson Diffusions**
  Ivan Papić
- **Boundary behaviour of layer potentials for the multi-term time-fractional diffusion equation**
  Karolina Pawlak
- **A convective bulk-surface Cahn-Hilliard model**
  Jonas Stange

Homogenization across scales: from nonlinear elliptic PDEs to amyloid-β transport in brain tissue

Miroslav Bulíček
Charles University, Czechia

Abstract: Homogenization links microscopic structures to effective macroscopic models. We illustrate this in two contexts—one theoretical, one biological. First, we consider strongly nonlinear elliptic problems with homogeneous Dirichlet conditions, where the growth and coercivity are governed by an inhomogeneous anisotropic N-function. Existence results are shown under theΔ_2-condition on the N-function or its convex conjugate, followed by homogenization for periodic families of such problems. Second, we apply homogenization to amyloid-β transport in cortical tissue. Here, linear equations with Newton-type boundary conditions on cells yield effective volume terms in the homogenized problem, bridging microscale dynamics with tissue-scale behavior and providing insight into neurodegenerative disease mechanisms.

Particle approximation for linear and fast diffusion equations

Antonio Esposito
University of L'Aquila, Italy

Abstract: In this talk I will discuss a possible way to construct deterministic particle solutions for linear and fast diffusion equations using a nonlocal approximation. The strategy is based on the 2-Wasserstein gradient flow structure of the equations in order to obtain the nonlocal approximating PDEs by regularising the corresponding internal energy with suitably chosen mollifying kernels, either compactly or globally supported.
This is a joint work with J. A. Carrillo (Oxford), J. Skrzeczkowski (Oxford), and J. S. H. Wu (Manitoba).

Aggregation-Diffusion Equations with saturation in bounded domains

David Gómez-Castro
Autonomous University of Madrid, Spain

Abstract: In this talk we will discuss the well-posedness, asymptotics, and numerical analysis of aggregation-diffusion equations with non-linear mobility

$$\partial_t \rho = \operatorname{div}(\mathsf{m}(\rho)\nabla(U'(\rho) + V + W * \rho)).$$

We are interested in non-linear mobilities of saturation type where $\mathsf{m}(0) = \mathsf{m}(\alpha) = 0$ and $0 \leqslant \rho_0 \leqslant \alpha$. We consider bounded domains and no-flux conditions. These are formally gradient flows of a modified Wasserstein-type distances. For the case $W = 0$ we will discuss an existence theory of semigroup solutions with comparison principle, and time-asymptotics. We will also discuss properties and convergence of numerical schemes of finite-volume type in the general setting based on discrete Sobolev and Wasserstein-type estimates.

The talk presents joint work with J.A. Carrillo and A. Fernández-Jiménez (U. Oxford)

On the singular limit of Brinkman's law to Darcy's law

Matt Jacobs
Purdue University, USA

Abstract: In this talk, I will discuss the singular limit of Brinkman's law to Darcy's law in the context of congestion driven motion models. These models arise in various scenarios, in particular, in the continuum description of cellular and tissue growth. Our result shows that different versions of these models are related and can be obtained from one another. The main ingredient of our analysis is a family of energy evolution equations and their dissipation structures, which are novel and of independent interest. This strategy allows us to establish our result for a much larger family of pressure laws than was previously possible in the literature. Furthermore, our analysis can handle the joint limit starting from a compressible Brinkman's model and converging to an incompressible Darcy's law model, where the latter is a Hele-Shaw type free boundary problem.

Fully nonlinear parabolic mean field games

Espen Jakobsen
Norwegian University of Science and Technology, Trondheim, Norway

Abstract: Existence and uniqueness results, and some degenerate problems
We introduce a class of fully-nonlinear parabolic Mean Field Games systems (PDEs) corresponding to Mean Field Games with controlled local and/or nonlocal diffusion. After a heuristic derivation, we will focus on 3 model problems: (i) Nondegenerate local 2nd order problems, (ii) Nondegenerate nonlocal problems (e.g. with fractional Laplacians), and (iii) a degenerate nonlocal problem. In all cases we give existence and uniqueness results and discuss proofs. Some highlights we may be able to touch upon: A moment free theory of MFGs, uniqueness of MFGs without strict convexity or strict monotonicity, existence for MFG without uniqueness for Fokker-Planck, uniqueness for a Fokker-Planck equation with degenerate non-Lipschitz coefficient via non-standard viscosity solution doubling of variables. This talk is based on joint work with Milosz Krupski (Wroclaw) and Indranil Chowdhury (Kanpur) contained in the two papers: SIMA 2024 https://epubs.siam.org/doi/epdf/10.1137/23M1615528 and JDE 2025 https://doi.org/10.1016/j.jde.2025.113436.

Convexity propagation and convex ordering of one-dimensional stochastic differential equations

Benjamin Jourdain
École des Ponts ParisTech, France

Abstract: We consider driftless one-dimensional stochastic differential equations. We first recall how they propagate convexity at the level of one-dimensional marginals. We show that for convexity propagation at the level of multidimensional marginals, some convexity of the diffusion coefficient is needed. We obtain functional convexity propagation under a slight renforcement of this necessary condition. For directional convexity propagation, no condition is needed.

Everything you always wanted to know about PDEs with frational-in-time derivateves, but were afraid to ask

Grzegorz Karch
University of Wrocław, Poland

Abstract: A linearization principle for abstract reaction-diffusion systems with fractional-in-time derivatives will be presented in the talk. This principle states that the stability (or instability) of a solution to a suitable linearization of a nonlinear problem implies the stability (or instability) of a solution to the original nonlinear problem. We proved that this principle in the fractional case is essentially the same as in the case of equations with classical (first-order) derivatives.
However, new phenomena can still be observed. Namely, fractional derivatives can stabilize stationary solutions in the sense that a stationary solution may be stable in the fractional setting even if it is unstable in the case of classical (first-order) derivatives.

Talk based on the preprint:
Sofwah Ahmad, Szymon Cygan, Grzegorz Karch, *Tools for stability analysis of fractional reaction diffusion systems*, (2025), arXiv:2507.02094 [math.AP]

Harnack estimates for nonlocal diffusions.

Naian Liao
University of Salzburg, Austria

Abstract: I will introduce new Harnack estimates for nonlocal diffusion equations that defy the waiting-time phenomenon – based on a joint work with Marvin Weidner. I will also report on Harnack estimates for nonlocal diffusion equations with a drift.

Local convergence and metastability for mean-field particles in a multi-well potential

Pierre Monmarché
Sorbonne University, Paris, France

Abstract: We consider particles following a diffusion process in a multi-well potential and attracted by their barycenter (corresponding to the particle approximation of the Wasserstein flow of a suitable free energy). It is well-known that this process exhibits phase transitions: at high temperature, the mean-field limit has a single stationary solution, the N-particle system converges to equilibrium at a rate independent from N and propagation of chaos is uniform in time. At low temperature, there are several stationary solutions for the non-linear PDE, and the limit of the particle system as N and t go to infinity do not commute. We show that, in the presence of multiple stationary solutions, it is still possible to establish local convergence rates for initial conditions starting in some Wasserstein balls (this is a joint work with Julien Reygner). In terms of metastability for the particle system, we also show that for these initial conditions, the exit time of the empirical distribution from some neighborhood of a stationary solution is exponentially large with N and approximately follows an exponential distribution, and that propagation of chaos holds uniformly over times up to this expected exit time (hence, up to times which are exponentially large with N).

Aronson-Bénilan estimates for the parabolic-elliptic Keller-Segel model

Filippo Santambrogio
Université de Lyon 1, France

Abstract: This talk, based on a joint work (still to be finished) with Charles Elbar (Lyon) and Alejandro Fernandez Jimenez (Oxford), lies at the intersection of two well-known phenomena in parabolic equations. The first concerns nonlinear estimates on the second derivatives of solutions to diffusion equations: by looking at the PDE satisfied by the Laplacian of the logarithm of the solution of the heat equation (or of suitable powers of the solution for non-linear diffusion such as porous media equations) one can obtain lower bounds in the form -C/t. The second, instead, concerns the critical mass in the parabolic-elliptic Keller-Segel chemotactic system where linear diffusion is coupled with advection by a potential generated by the convolution of the solution with the Poisson Kernel: it is well-known for this nonlinear equation that explosion in finite time or global existence depends on the mass (which is preserved in time) and the best estimates are obtained for small mass. In the talk I will show how, under small mass assumptions, it is also possible to obtain estimates, with instantaneous regularization, on the Laplacian of the pressure for the critical case, i.e. for the case of linear diffusion in 2D where the pressure is logarithmic or in power-case in higher dimension. I will also explain how to deal with the case of subcritical masses (not necessarily very small), and explain that the notion of critical mass is also related to the minimal mass of subsolutions of a certain nonlinear elliptic equation (Liouville or Lane-Emden). I will most likely focus on the 2D case, easier to explain, and briefly address the modifications to deal with the higher dimensional case.

Doubly nonlinear parabolic systems in time-dependent domains

Christoph Scheven
University of Duisburg–Essen, Germany

Abstract: The talk deals with the Cauchy-Dirichlet problem for doubly nonlinear parabolic systems on non-cylindrical domains in space-time. In other words, we consider spatial domains whose shape varies in time. The equations considered include parabolic p-Laplace equations and porous medium type equations as special cases. In the case of a growing domain, the boundary values can be interpreted as additional initial conditions, while in the case of a shrinking domain, the boundary values can be seen as a kind of obstacle condition. The treatment of time-varying domains turns out to be significantly harder than the standard case of cylindrical domains. We present an existence result for solutions to such problems under very weak regularity assumptions on the domain. A first regularity result for the constructed solutions guarantees that they depend continuously on time with respect to the L^2-norm if the domain does not shrink too fast.

Degenerate cross-diffusion as the inviscid limit of nonlocal tissue growth models

Markus Schmidtchen
Technical University Dresden, Germany

Abstract: This talk will present three recent results on nonlinear diffusion systems motivated by models in biology. We begin with multiphase tissue models where velocity–pressure relations are governed by Brinkman or Darcy laws. We show that solutions of viscoelastic Brinkman systems converge to weak solutions of inviscid Darcy models as viscosity vanishes, thereby providing a rigorous bridge between these classical frameworks. The second part turns to tissue growth with multiple phenotypic subpopulations and we examine coupled limits in viscosity and the number of species. Finally, we discuss anisotropic cross-diffusion systems describing tissue in heterogeneous media. Here, we establish the convergence of nonlocal approximations to their local degenerate parabolic limits.
This talk is based on joint works with N. David, T. Dębiec, and M. Mandal.

Long-time analysis of second-order Langevin diffusions with distribution-dependent forces and their numerical discretizations

Katharina Schuh
TU Wien, Austria

Abstract: In this talk, we explore the long-time behaviour of both the classical second-order Langevin diffusion and a non-linear variant with distribution-dependent forces of McKean-Vlasov type. In addition, we study a class of kinetic Langevin sampler – numerical discretization schemes for these continuous dynamics – and investigate their asymptotic behaviour. We establish L^1 Wasserstein contraction for both the continuous dynamics and its numerical approximations using couplings and provide qualitative error bounds for the proposed numerical schemes.
Furthermore, in the non-linear case, we exploit the connection to the cor- responding particle system and we present a uniform in-time propagation of chaos result in L^1 Wasserstein distance, as well as a quantitative result of strong convergence in relative entropy for a specific kinetic Langevin sampler.

Landis type results for discrete partial differential equations

Diana Stan
University of Cantabria, Spain

Abstract: We will present some Landis-type results for both the semidiscrete heat and the stationary discrete Schrödinger equations. For the semidiscrete heat equation we show that, under the assumption of two-time spatial decay conditions on the solution u, then necessarily $u \equiv 0$. For the stationary discrete Schrödinger equation we deduce that, under a vanishing condition at infinity on the solution u, then $u \equiv 0$. In order to obtain such results, we demonstrate suitable quantitative upper and lower estimates for the L2-norm of the solution within a spatial d-dimensional lattice of step h¿0. These estimates manifest an interpolation phenomenon between continuum and discrete scales.
This work is in collaboration with Aingeru Fernández Bertolin and Luz Roncal.

Borderline regularity in singular free boundary problems

José Miguel Urbano
KAUST, Saudi Arabia and University of Coimbra, Portugal

Abstract: We address the borderline regularity of local minimizers of energy functionals under minimal assumptions on the potential term σ. When σ is merely bounded and measurable, we show that sign-changing minimizers are Log-Lipschitz continuous, which is optimal in this general setting. In the one-phase case, however, we obtain gradient bounds along the free boundary, revealing a structural gain in regularity. Most notably, we prove that if σ is continuous, then minimizers are of class C^1 along the free boundary, thereby identifying a sharp threshold for differentiability in terms of the regularity of the potential. This is joint work with D. Araújo (UFPB), A. Sobral (KAUST), and E. Teixeira (OSU).

Mass Conservation in nonlinear diffusion

Juan Luis Vazquez
Autonomous University of Madrid, Spain

Abstract: In a survey paper we examine the validity of the principle of mass conservation (MC) for solutions of some typical equations in the theory of nonlinear diffusion, including equations in standard differential form and also their fractional counterparts. Starting from the well-kown MC for the heat equation, we use as main examples the porous medium equation and the p-Laplacian equation. Though these equations have the form of conservation laws, it happens that in some ranges of exponents the solutions posed in the whole Euclidean space actually lose mass in time. From the start we pay attention to the close connection between the validity of mass conservation and the existence of finite-mass self-similar solutions, as well as their role in fixing the asymptotic behaviour of more general classes of solutions. The study includes in recent times nonlinear equations with fractional Laplacian diffusion, doubly nonlinear equations, anisotropic diffusion, and several other topics of current interest. Some open problems are solved and others are posed

Short talks

Evolution variational inequalities with general costs

Pierre-Cyril Aubin-Frankowski
ENPC, IP Paris, France

Abstract: How to go beyond the gradient flows in metric spaces of the green book? I will present evolution variational inequalities (EVIs) driven by general cost functions c, which include Bregman and entropic transport divergences. Despite not having a metric, several properties of the resulting flows, including stability and energy identities still hold. Using novel notions of convexity related to costs c, I will show that EVI flows are the limit of splitting schemes, providing assumptions for both implicit and explicit iterations. Based on https://arxiv.org/abs/2505.00559, a joint work with Giacomo Enrico Sodini and Ulisse Stefanelli (UniVienna)

Fully Explicit Polynomial Convergence Rates of a Numerical Scheme for Aggregation–Diffusion Equations with Slow Diffusion

Matej Benko
Brno University of Technology, Czechia

Abstract: We propose a novel splitting algorithm with fully discrete in time and space explicit polynomial convergence rates for systems of nonlinear diffusion equations involving potential and interaction energies. This is possible due to the use of a localized JKO scheme combined with the finite-difference scheme. Unlike existing methods that rely on the propagation of chaos with exponential rates, our approach ensures efficient and accurate simulations of interacting particle systems. Theoretical and numerical results confirm its effectiveness for computational physics applications. This talk is based on joint work with Iwona Chlebicka and Błażej Miasojedow from University of Warsaw and Jørgen Endal from Norwegian University of Science and Technology (NTNU) in Trondheim.

Zolotarev distance, optimal elastic systems, and a pair of diffusions

Karol Bołbotowski
Warsaw University of Technology, Poland,

Abstract: The second Zolotarev distance between a pair of probabilities on $\mathbb{R}^n$ is expressed through a variational formulation with local bounds on the Hessian. It admits the dual problem whose solutions encode optimal configurations of elastic systems governed by a second-order PDE. In my talk I will show that at the foundation of these problems lies a pair of diffusive processes with the respective probabilities as the initial marginal laws and with a common, carefully selected terminal law.
This is a joint work with Guy Bouchitté (Université de Toulon).

Nodal sets of supersolutions to Schrödinger equations

Tomasz Klimsiak
Nicolaus Copernicus University, Toruń, Poland

Abstract: We address the question posed by H. Brezis concerning the structure of the set $\{u = 0\}$ for non-negative supersolutions to the equation

$$-Lu + Vu = 0 \quad \text{in } E, \tag{1}$$

where V is a singular potential on E and L is a self-adjoint linear operator on $L^2(E; m)$ related to a regular symmetric Dirichlet form $(\mathcal{E}, \mathcal{D}(\mathcal{E}))$. The class of admissible potentials V consists of positive *smooth measures*, which includes, in particular, locally quasi-integrable positive functions, as well as *generalized potentials*, i.e. positive Borel measures that may be concentrated on m-negligible sets. Using the Green function of L, we characterize the minimal set - depending only on L and V - where all possible zeros of non-trivial supersolutions to (1) must lie. The key ingredient in establishing this structure result is the Feynman-Kac-type representation for supersolutions to (1). As a corollary, we provide a necessary and sufficient condition on the potential V, under the sole assumption that $V : \mathbb{E} \to [0, \infty]$ is Borel measurable, ensuring that the strong maximum principle holds for the operator $-L + V$.

We present the results reported in Klimsiak, T.: Location of zeros of non-trivial positive supersolutions to Schrödinger equations. *Math. Ann.* **392** (2025)

Parareal in time and spectral in space fast L1 quasilinear subdiffusion solver

Łukasz Płociniczak
Wrocław University of Science and Technology, Poland

Abstract: We study an initial–boundary value problem for a quasilinear time-fractional diffusion equation of order $0 < \alpha < 1$, and propose a fully discrete numerical method that combines the parareal time-parallel algorithm with an L1 finite-difference scheme for the Caputo derivative and a spectral Galerkin method for spatial discretization. The primary contribution of this work is the first rigorous convergence analysis of the parareal–L1 method in the context of nonlinear subdiffusion. By constructing appropriate energy norms and using the orthogonality properties of the spectral basis, we prove that the parareal iterations converge exactly to the fully sequential L1–spectral solution within a finite number of steps, with convergence rates that do not depend on the fractional order. The spectral spatial discretization provides exponential convergence in space, while the parareal framework enables a speedup in computational time proportional to the number of processors used.
Supported by Polish National Agency for Academic Exchange (NAWA) under the Bekker Programme with the signature BPN/BEK/2024/1/00002.

Schauder estimates for viscous Hamilton–Jacobi equations with nonlocal operators

Artur Rutkowski
Wrocław University of Science and Technology, Poland

Abstract: We give Schauder estimates and well-posedness results for subcritical viscous Hamilton–Jacobi equations under a mild order condition for the viscosity term, expressed via its heat kernel. We also investigate blow-up rates of various Hölder norms of solutions for small times when the initial condition is of low regularity, including cases where the gradient becomes unbounded. Based on a joint work with Espen Jakobsen and Robin Lien (NTNU, Trondheim).

From indirect to direct taxis by fast reaction limit

Dariusz Wrzosek
University of Warsaw, Poland

Abstract: The asymptotic transition between two parabolic quasi-liner models of chemotaxis which can be viewed as a fast reaction limit is studied . The first of them refers to a chemical signaling, mechanism ubiquitous in nature, consisting of the response of individuals of one population to the gradient of a chemical substance secreted by individuals of the other (indirect taxis). The second refers to the direct reaction to the gradient of population density, i.e. direct taxis, in this context. The kinetic part of the models describe the competition and the taxis term refers to the avoidance strategy attributed to one of the competitors. The talk is related to a joint work with Jose Ignacio Tello from Madrid.