Book of Abstracts

Hydrodynamic models and multi-scale analysis in PDEs

Institute of Mathematics of Polish Academy of Sciences
ul. Śniadeckich 8, 00-656 Warszawa, Poland
14–17 September 2025

Organizers: Iwona Chlebicka, Tomasz Piasecki, Aneta Wróblewska-Kamińska

List of participants

1. **Nilasis Chaudhuri**, University of Warsaw, Poland
2. **Young-Pil Choi**, Yonsei University, Korea
3. **Thomas Eiter**, Weierstrass Institute for Applied Analysis and Stochastics, Germany
4. **Michał Fabisiak**, University of Warsaw, Poland
5. **Francesco Fanelli**, BCAM, Spain
6. **Jinwook Jung**, Hanyang University, Korea
7. **Roman Korsak**, Polish Academy of Sciences, Poland
8. **Rober Lazarzik**, Weierstrass Institute for Applied Analysis and Stochastics, Germany
9. **Michał Łasica**, Polish Academy of Sciences, Poland
10. **Muhammed Ali Mehmood**, Imperial College London, UK
11. **Šarká Nečasová**, Czech Academy of Sciences, Czechia
12. **Václav Mácha**, Czech Academy of Sciences, Czechia
13. **Piotr Mucha**, University of Warsaw, Poland
14. **Ondřej Kreml**, Czech Academy of Sciences, Czechia
15. **Florian Oschmann**, Czech Academy of Sciences, Poland
16. **Jan Peszek**, University of Warsaw, Poland
17. **Katarzyna Ryszewska**, Polish Academy of Sciences, Poland
18. **Jonas Stange**, University of Regensburg, Germany
19. **Maja Szlenk**, Université Savoie Mont Blanc, France
20. **Jakub Woźnicki**, University of Warsaw, Poland
21. **Ewelina Zatorska**, Warwick University, UK

Organizers

1. **Iwona Chlebicka**, University of Warsaw, Poland
2. **Tomasz Piasecki**, University of Warsaw, Poland, `tpiasecki@mimuw.edu.pl`
3. **Aneta Wróblewska-Kamińska**, Polish Academy of Sciences, Poland, `awrob@impan.pl`

Acknowledgments

The workshop *Hydrodynamic models and multi-scale analysis in PDEs* is supported by University of Warsaw in frame Thematic Research Programme: *Evolutionary PDEs 2025* and by Institute of Mathematics Polish Academy of Sciences and the National Science Centre Poland with Sonata Bis project: *Mathematical analysis of hydrodynamic models nonlinearities, non-locality, domain, scales* (2020/38/E/ST1/00469).

Analysis for hydrodynamic model of swarming

Nilasis Chaudhuri

University of Warsaw, Poland
NCHAUDHURI@MIMUW.EDU.PL

Abstract

In this talk, we consider a one-dimensional hydrodynamic model featuring nonlocal attraction-repulsion interactions and singular velocity alignment. We introduce a two-velocity reformulation and derive a corresponding energy-type inequality, in the spirit of the Bresch–Desjardins estimate. Furthermore, we identify dependencies between the communication weight and the interaction kernel, as well as between the pressure and the viscosity term, which allow this inequality to be uniform in time. This framework is then used to analyze the long-time asymptotics of solutions.

References

[1] N Chaudhuri, YP Choi, O Tse, E Zatorska. Existence of weak solutions and long-time asymptotics for hydrodynamic model of swarming *Journal of the London Mathematical Society 111 (2), e70088.*

Exponential and algebraic decay in Euler–alignment system with nonlocal interaction forces

Young-Pil Choi

Yonsei University, Republic of Korea
YPCHOI@YONSEI.AC.KR

Abstract

In this talk, we study the large-time behavior of the pressureless Euler system with nonlocal alignment and interaction forces. For general interaction potentials and communication weights, we obtain quantitative convergence of classical solutions. In 1D, (λ, Λ)-convex potentials yield exponential decay for bounded weights and sharp algebraic rates for weakly singular ones. For the Coulomb–quadratic potential, we prove exponential convergence with bounded weights and polynomial bounds with singular ones. In multi-dimensions, uniformly (λ, Λ)-convex potentials give exponential or improved algebraic decay depending on the weight. In all cases, the density converges (up to translation) to the interaction energy minimizer, while velocities align to a constant. Our results highlight that convergence rates are determined solely by the local behavior of the communication kernel: bounded weights produce exponential decay, while weakly singular ones yield algebraic rates.

Long-time behavior of the damped Euler equations with direction dependent spatial limits

Thomas Eiter

Freie Universität Berlin & Weierstrass Institute Berlin, Germany
THOMAS.EITER@WIAS-BERLIN.DE

Abstract

Consider the Euler equations for isentropic flow with frictional damping in the whole space. If we prescribe the density values at spatial infinity by a direction-dependent function, then solutions develop a non-trivial self-similar pattern as $t \to \infty$. More precisely, we show the convergence of the density towards a self-similar solution to the porous medium equation while the associated limit momentum is governed by Darcy's law. To this end, we transform the system into parabolic scaling variables and derive convergence rates in terms of a relative entropy. The analysis is performed in a framework of energy-variational solutions since, in contrast to weak solutions, their existence can be established in multiple dimensions.

References

[1] T. Eiter and S. Schindler. Time-asymptotic self-similarity of the damped compressible Euler equations in parabolic scaling variables. arXiv:2507.03688

On heat conducting fluids: incompressible limit, fast rotation and the Rayleigh-Bénard problem

Francesco Fanelli

Basque Center for Applied Mathematics, Spain
FFANELLI@BCAMATH.ORG

Abstract

In this talk, we revisit some recent results about the dynamics of heat conducting fluids in the incompressible limit regime, under non-homogeneous boundary conditions for the temperature (a relevant situation in the context of the celebrated Rayleigh-Bénard problem). We show how the classical Oberbeck-Boussinesq system, usually identified as the target system, must be modified in this case, or even dismissed in presence of additional fast rotation effects.

On the temporal estimates for the incompressible Navier-Stokes equations and Hall-magnetohydrodynamic equations

Jinwook Jung

Hanyang University, Republic of Korea
jinwookjung@hanyang.ac.kr

Abstract

In this talk, we discuss decay rates of the solutions to the incompressible Navier-Stokes equations and Hall-magnetohydrodynamic equations. We first present the improved decay rate of weak solutions of these equations by refining the Fourier splitting method with initial data in the space of pseudo-measures. We also deal with these equations with initial data in Lei-Lin spaces and find decay rates of solutions in Lei-Lin spaces. This talk is based on the collaboration with H. Bae (UNIST) and J. Shin (Yonsei University).

On dissipative turbulent solutions to the compressible anisotropic Navier-Stokes equations in unbounded domains

Ondřej Kreml

Institute of Mathematics, Czech Academy of Sciences, Czechia
kreml@math.cas.cz

Abstract

Inspired by Abbatiello, Feireisl and Novotný (1), we prove the global existence of dissipative turbulent solution for the compressible Navier-Stokes equations with anisotropic viscous stress tensor on unbounded domain. Our work complements the result of Bresch and Jabin (2), where the authors used the new compactness method to prove the existence of a weak solution to the same system in T^3. By working with larger class of dissipative turbulent solutions, we are able to relax assumptions on the anisotropic tensor coefficients and the pressure law coefficient and we establish the existence result on a large class of unbounded domains, which is more conform to physical context. We also prove the weak-strong uniqueness property of acquired dissipative turbulent solutions.

This is a joint work with Šárka Nečasová and Tong Tang.

References

[1] A. Abbatiello, E. Feireisl, and A. Novotný, Generalized solutions to models of compressible viscous fluids, *Discrete Contin. Dyn. Syst.* , 41 (2021), 1–28.

[2] D. Bresch and P. E. Jabin, Global existence of weak solutions for compressible Navier-Stokes equations: thermodynamically unstable pressure and anisotropic viscous stress tensor, *Ann. of Math.*, 188 (2018), 577–684.

Energy-variational framework in fluid dynamics

Robert Lasarzik

WIAS Berlin, Germany
LASARZIK@WIAS-BERLIN.DE

Abstract

We introduce the concept of energy-variational solutions for a general class of evolutionary partial differential equations. This was already introduced for abstract evolution equations (1) and especially conservation laws (2). The two guiding examples that we will consider in this talk in order to generalize the existing results are the Euler—Korteweg equations and the binormal curvature flow. Focusing on these examples, we highlight the key assumptions in the abstract setting. We will comment on the proof of existence via a novel minimizing movement scheme and discuss possible selection criteria. This is a joint work with Thomas Eiter and Marcel Śliwiński (3).

References

[1] A. Agosti, R. Lasarzik, and E. Rocca. Energy-variational solutions for viscoelastic fluids. *Adv. Nonlinear Anal.*, 13, 2024. https://doi.org/10.1515/anona-2024-0056.

[2] T. Eiter and R. Lasarzik. Existence of energy-variational solutions to hyperbolic conservation laws. *Calc. Var.* 63:103, 2024. https://doi.org/10.1007/s00526-024-02713-9.

[3] T. Eiter, R. Lasarzik, and M. Śliwiński. Energy-variational framework in fluid dynamics — existence and selection. *in preparation* 2025.

A variational framework for singular limits of gradient flows

Michał Łasica

IMPAN, Poland
MLASICA@IMPAN.PL

Abstract

A lower semicontinuous, convex functional $\mathcal{E}\colon X \to [0,\infty]$ with dense domain on a Hilbert space X generates a gradient flow S whose trajectories are the unique, continuous, globally existing solutions to

$$\tfrac{\mathrm{d}u}{\mathrm{d}t} \in -\partial\mathcal{E}(u(t)) \text{ for } t>0, \quad u(0)=u_0 \in X. \tag{$*$}$$

It is known that for a family $\mathcal{E}_\varepsilon\colon X \to [0,\infty]$, $\varepsilon \geq 0$ of such functionals, uniform convergence of trajectories of S_ε to trajectories of S_0 as $\varepsilon \to 0^+$ is equivalent to the Mosco convergence of $\mathcal{E}_\varepsilon$ to $\mathcal{E}_0$. This allows one to understand the limit passage without performing any analysis of the equation $(*)$; actually, without any knowledge about the subdifferentials $\partial\mathcal{E}_\varepsilon$.

However, in many applications, such as thin domains, boundary layers, or discrete-to-continuum limits, the gradient flows are defined on different (if often isomorphic) Hilbert spaces X_ε, with a singular change in the metric occurring as $\varepsilon \to 0^+$. When considering limits of stationary variational problems, one can usually deal with this hurdle by embedding X_ε, $\varepsilon \geq 0$ in a common Hilbert space. However, this is less obvious in the case of gradient flows, where the evolution is implicitly defined in terms of the metric.

In order to systematically deal with this issue, we introduce a notion of Mosco convergence of spaces X_ε and functionals $\mathcal{E}_\varepsilon$ along a given family of *connecting operators* $L_\varepsilon\colon X_\varepsilon \to X_0$, which generalizes the usual Mosco convergence. We show that this convergence implies the uniform convergence of trajectories (in a suitable sense defined using operators L_ε). We give examples of application of this framework to various problems involving the parabolic p-Laplace equation, $p \geq 1$.

This is joint work with Y. Giga (UTokyo) and P. Rybka (UWarsaw).

Non-Newtonian compressible fluids with white noise

Vaclav Macha

Institute of Mathematics, Czech Academy of Sciences, Czech Republic
macha@math.cas.cz

Abstract

We discuss the existence of the Young measures for random processes. Subsequently, this tool is used to deliver the existence result for the generalized compressible Navier-Stokes equation with shear-rate dependent viscosity.

Weak solutions to a full compressible magnetohydrodynamic flow interacting with thermoelastic structure

Šárka Nečasová

Institute of Mathematics, Czech Academy of Sciences, Czech Republic
matus@math.cas.cz

Abstract

This lecture is devoted to the problem interaction between a full compressible, electrically conducting fluid and a thermoelastic shell in a two-dimensional setting. The shell is modeled by linear thermoelasticity equations, and encompasses a time-dependent domain which is filled with a fluid described by full compressible (non-resistive) magnetohydrodynamic equations. The magnetohydrodynamic flow and the shell are fully coupled, resulting in a fluid-structure interaction problem that involves heat exchange. We establish the existence of weak solutions through *domain extension*, *operator splitting*, *decoupling*, *penalization of the interface condition*, and *appropriate limit passages*. It is a join work with Kuntal Bhandari and Bingkang Huang, see [1].

References

[1] Kuntal Bhandari, Bingkang Huang, Šárka Nečasová: Weak solutions to a full compressible magnetohydrodynamic flow interacting with thermoelastic structure, arXiv:2505.23539

Per aquam ad astra: on stars (and) boiling water

Florian Oschmann

Czech Academy of Sciences, Mathematical Institute, Czech republic
OSCHMANN@MATH.CAS.CZ

Abstract

We will derive the so-called (magneto-)Oberbeck-Boussinesq approximation from the compressible (MHD-)Navier-Stokes-Fourier system by means of singular limits. This approximation tells that, in a certain regime, the density deviations can be approximated linearly by temperature deviations. Instead of taking Neumann boundary values for the temperature, Dirichlet boundary conditions are imposed, giving rise to an additional non-local forcing term in the limiting heat equation. This is joint work with Peter Bella (TU Dortmund), Eduard Feireisl (CAS), Piotr Gwiazda (PAN) and Aneta Wróblewska-Kamińska (PAN).

References

[1] Bella, P.; Feireisl, E.; Oschmann, F.: Rigorous derivation of the Oberbeck–Boussinesq approximation revealing unexpected term. *Communications in Mathematical Physics*, 403.3 (2023): 1245-1273.

[2] Gwiazda, P.; Oschmann; F., Wróblewska-Kamińska, A.: Rigorous derivation of magneto-Oberbeck-Boussinesq approximation with non-local temperature term. *arXiv preprint arXiv:2504.13525* (2025).

Hydrodynamic limit of singular alignment

Jan Peszek

J.PESZEK@MIMUW.EDU.PL

Abstract

One of the key goals in deriving hydrodynamic limits of multi-agent systems is to uncover interesting and useful macroscopic properties of their dynamics. In the case of classical hard-sphere interactions, the transition from a particle system to the Boltzmann equation, and eventually to the Navier-Stokes equations, leads to a significant simplification of the underlying chaotic dynamics. In this talk, I will present the natural hydrodynamic limit of the Cucker–Smale flocking model with a strongly singular communication weight - namely, the pressureless fractional Euler-alignment system - and address the question: what macroscopic properties emerge due to the singular nature of the interactions?

Global existence results for compressible non-Newtonian fluids

Maja Szlenk

Université Savoie Mont Blanc

Abstract

My talk will be based on the recent results obtained in collaboration with Didier Bresch (USMB) and Cosmin Burtea (Université Paris-Cité). The first part will be devoted to the existence of weak solutions of a steady compressible Navier–Stokes system for a power-law fluid.

In the second part, I will present the results concerning the so-called "thick fluids", arising as a limit of a power-law fluid when $p \to \infty$. In the limit we obtain a two-phase system with a unilateral constraint on the strain tensor. For this model, we derive the existence of variational weak solutions in the semi-stationary case, and extend the result to a full Navier–Stokes system in 1D.

Recent advances in the modeling of the electrorheological fluids

Jakub Woźnicki

University of Warsaw; Polish Academy of Sciences, Poland
jw.woznicki@student.uw.edu.pl

Abstract

The modeling of electrotheological fluids has long been done by including a non-standard growth assumption on the stress tensor in the momentum equation. That is, one supposes

$$S(t,x,Du) \sim (\nu_0 + \nu_1 \, |Du|^{s(t,x)-2}) \, Du,$$

where s depends on the ambient electric field. Because of the properties of variable-exponent spaces $L^{s(t,x)}$, which replace the standard Lebesgue spaces in this setting, much of the research has focused on exponents that exhibit a certain smoothness with respect to the space and time variables. This requirement was a significant restriction, since in principle the electric field may be discontinuous in time.

Recently, however, a new technique has been developed that allows one to drop any time-regularity assumptions, requiring only measurability. In this talk, we present results on the existence and uniqueness of various notions of solutions to the incompressible fluid equations in bounded domains, as well as to general parabolic equations with monotone operators satisfying the non-standard growth condition.

Mathematical Analysis of Multi-Phase Flow Models

Ewelina Zatorska

University of Warwick, UK
ewelina.zatorska@warwick.ac.uk

Abstract

I will discuss models of multi-component multi-phase compressible flows, their derivation, analytical challenges, partial results that we obtained so far, as well as open problems. At the end of my talk i will reveal why these systems are interesting for someone working on collective behaviour models.